Package ‘ksm’

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Type Package

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BugReports `https://github.com/lbelzile/ksm/issues`

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`bandwidth_optim` *Bandwidth optimization for symmetric matrix kernels*

Description

Given a sample of positive definite matrices, perform numerical maximization of the `h`-block least square (`lscv`) or leave-one-out likelihood (`lcv`) cross-validation criteria using a root search.

Usage

```
bandwidth_optim(
  x,
  criterion = c("lscv", "lcv"),
  kernel = c("Wishart", "smlnorm", "smnorm"),
  tol = 1e-04,
  h = 1L,
  bounds = c(1e-04, 10)
)
```

Arguments

`x`	sample of symmetric matrix observations from which to build the kernel density kernel
`criterion`	optimization criterion, one of `lscv` for least square cross-validation at lag `h` or `lcv` for leave-one-out cross-validation.
`kernel`	string, one of `Wishart`, `smlnorm` (log-Gaussian) or `smnorm` (Gaussian).
`tol`	double, tolerance of optimization (root search)
`h`	lag step for consideration of observations, for the case `criterion=lscv`
`bounds`	vector of length 2 containing the bounds for the search

Value

double, the optimal bandwidth up to `tol`

Examples

```
x <- simu_rdens(n = 100, model = 3, d = 3)
bandwidth_optim(x = x,
 criterion = "lscv",
 kernel = "Wishart",
 h = 2L)
```

`dinvWishart` *Density of inverse Wishart random matrix*

Description

Density of inverse Wishart random matrix

Usage

```
dinvWishart(x, df, S, log = FALSE)
```

Arguments

`x`	array of dimension `d` by `d` by `n`
`df`	degrees of freedom
`S`	symmetric positive definite matrix of dimension `d` by `d`
`log`	logical; if `TRUE`, returns the log density

Value

a vector of length `n` containing the log-density of the inverse Wishart.

`dmbeta2` *Matrix beta type II density function*

Description

Given a random matrix `x`, compute the density for arguments `shape1` and `shape2`

Usage

```
dmbeta2(x, shape1, shape2, log = TRUE)
```

Arguments

`x`	cube of dimension `d` by `d` by `n` containing the random matrix samples
`shape1`	positive shape parameter, strictly larger than $(d-1)/2$.
`shape2`	positive shape parameter, strictly larger than $(d-1)/2$.
`log`	[logical] if `TRUE` (default), returns the log density.

Value

a vector of length `n`

`dsmlnorm` *Symmetric matrix-variate lognormal density*

Description

Density of the lognormal matrix-variate density, defined through the matrix logarithm, with the Jacobian resulting from the transformation

Usage

```
dsmlnorm(x, b, M, log = TRUE)
```

Arguments

`x`	[cube] array of dimension `d` by `d` by `n`
`b`	[numeric] scale parameter, strictly positive
`M`	[matrix] location matrix, positive definite
`log`	[logical] if `TRUE` (default), returns the log density

Value

a vector of length `n`

`dsmnorm` *Symmetric matrix-variate normal density*

Description

Symmetric matrix-variate normal density

Usage

```
dsmnorm(x, b, M, log = TRUE)
```

Arguments

`x`	[cube] array of dimension `d` by `d` by `n`
`b`	[numeric] scale parameter, strictly positive
`M`	[matrix] location matrix, positive definite
`log`	[logical] if `TRUE` (default), returns the log density

Value

a vector of length `n`

`dWishart` *Density of Wishart random matrix*

Description

Density of Wishart random matrix

Usage

```
dWishart(x, df, S, log = FALSE)
```

Arguments

`x`	array of dimension `d` by `d` by `n`
`df`	degrees of freedom
`S`	symmetric positive definite matrix of dimension `d` by `d`
`log`	logical; if `TRUE`, returns the log density

Value

a vector of length `n` containing the log-density of the Wishart.

`integrate_spd` *Integration with respect to symmetric positive definite matrices*

Description

Given a function `f` defined over the space of symmetric positive definite matrices, compute an integral via numerical integration using the routine cubintegrate.

Usage

```
integrate_spd(
  f,
  dim,
  tol = 0.001,
  lb = 1e-08,
  ub = Inf,
  neval = 1000000L,
  method = c("suave", "hcubature"),
  ...
)
```

Arguments

`f`	function to evaluate that takes as arguments array of size `dim` by `dim` by 1.
`dim`	dimension of integral, only two or three dimensions are supported
`tol`	double for tolerance of numerical integral
`lb`	lower bound for integration range of eigenvalues
`ub`	upper bound for integration range of eigenvalues
`neval`	maximum number of evaluations
`method`	string indicating the method from `cubature`
`...`	additional arguments for the function `f`

Value

list returned by the integration routine. See the documentation of cubintegrate for more details.

Examples

```
integrate_spd(
  dim = 2L,
  neval = 1e4L,
  f = function(x, S){
   dWishart(x, df = 10, S = S, log = FALSE)},
  S = diag(2))
```

`kdens_smlnorm` *Symmetric matrix log-normal kernel density*

Description

Given a sample of `m` points `xs` from an original sample and a set of `n` new sample matrices `x` at which to evaluate the symmetric matrix normal log kernel, return the density with bandwidth parameter `b`.

Usage

```
kdens_smlnorm(x, xs, b, log = TRUE)
```

Arguments

`x`	cube of size `d` by `d` by `n` of points at which to evaluate the density
`xs`	cube of size `d` by `d` by `m` of sample matrices which are used to construct the kernel
`b`	positive double giving the bandwidth parameter
`log`	bool; if `TRUE`, return the log density

Value

a vector of length `n` containing the (log) density of the sample `x`

`kdens_smnorm` *Symmetric matrix normal kernel density*

Description

Given a sample of `m` points `xs` from an original sample and a set of `n` new sample matrices `x` at which to evaluate the symmetric matrix normal kernel, return the density with bandwidth parameter `b`. Note that this kernel suffers from boundary spillover.

Usage

```
kdens_smnorm(x, xs, b, log = TRUE)
```

Arguments

`x`	cube of size `d` by `d` by `n` of points at which to evaluate the density
`xs`	cube of size `d` by `d` by `m` of sample matrices which are used to construct the kernel
`b`	positive double giving the bandwidth parameter
`log`	bool; if `TRUE`, return the log density

Value

a vector of length `n` containing the (log) density of the sample `x`

`kdens_symmat` *Kernel density estimators for symmetric matrices*

Description

Given a sample of `m` points `xs` from an original sample and a set of `n` new sample symmetric positive definite matrices `x` at which to evaluate the kernel, return the density with bandwidth parameter `b`.

Usage

```
kdens_symmat(x, xs, kernel = "Wishart", b = 1, log = TRUE)
```

Arguments

`x`	cube of size `d` by `d` by `n` of points at which to evaluate the density
`xs`	cube of size `d` by `d` by `m` of sample matrices which are used to construct the kernel
`kernel`	string, one of `Wishart`, `smnorm` or `smlnorm`.
`b`	positive double giving the bandwidth parameter
`log`	bool; if `TRUE`, return the log density

Value

a vector of length `n` containing the (log) density of the sample `x`

`kdens_Wishart` *Wishart kernel density*

Description

Given a sample of `m` points `xs` from an original sample and a set of `n` new sample matrices `x` at which to evaluate the Wishart kernel, return the density with bandwidth parameter `b`.

Usage

```
kdens_Wishart(x, xs, b, log = TRUE)
```

Arguments

`x`	cube of size `d` by `d` by `n` of points at which to evaluate the density
`xs`	cube of size `d` by `d` by `m` of sample matrices which are used to construct the kernel
`b`	positive double giving the bandwidth parameter
`log`	bool; if `TRUE`, return the log density

Value

a vector of length `n` containing the (log) density of the sample `x`

`lcv_kdens_symmat` *Likelihood cross-validation for symmetric positive definite matrix kernels*

Description

Given a cube of sample observations (consisting of random symmetric positive definite matrices), and a vector of candidate bandwidth parameters `b`, compute the leave-one-out likelihood cross-validation criterion and return the bandwidth among the choices that maximizes the criterion.

Usage

```
lcv_kdens_symmat(x, b, h = 1L, kernel = "Wishart")
```

Arguments

`x`	array of dimension `d` by `d` by `n`
`b`	vector of candidate bandwidth, strictly positive
`h`	integer for the lag vector for determining which observation to exclude, any data point in a radius of `h`
`kernel`	string indicating the kernel, one of `Wishart`, `smlnorm` or `smnorm`.

Value

a list with arguments

- `lcv` vector of likelihood cross validation criterion
- `b` vector of candidate bandwidth
- `h` lag for leave-one-out
- `bandwidth` optimal bandwidth among candidates
- `kernel` string indicating the choice of kernel function

`lcv_kern_smlnorm` *Likelihood cross validation criterion for symmetric matrix lognormal kernel*

Description

Given a cube `x` and a bandwidth `b`, compute the leave-one-out cross validation criterion by taking out a slice and evaluating the kernel at the holdout value, excluding points that are at distance at least `h-1` apart.

Usage

```
lcv_kern_smlnorm(x, b, h = 1L)
```

Arguments

`x`	[cube] array of dimension `d` by `d` by `n`
`b`	[numeric] scale parameter, strictly positive
`h`	integer lag for excluding observations

Value

the value of the log objective function

`lcv_kern_smnorm` *Likelihood cross validation criterion for symmetric matrix normal kernel*

Description

Given a cube `x` and a bandwidth `b`, compute the leave-one-out cross validation criterion by taking out a slice and evaluating the kernel at the holdout values, excluding points that are at distance at least `h-1` apart.

Usage

```
lcv_kern_smnorm(x, b, h = 1L)
```

Arguments

`x`	[cube] array of dimension `d` by `d` by `n`
`b`	[numeric] scale parameter, strictly positive
`h`	integer lag for excluding observations

Value

the value of the log objective function

`lcv_kern_Wishart` *Likelihood cross validation criterion for Wishart kernel*

Description

Given a cube `x` and a bandwidth `b`, compute the leave-one-out cross validation criterion by taking out a slice and evaluating the kernel at the holdout value, excluding points that are at distance at least `h-1` apart.

Usage

```
lcv_kern_Wishart(x, b, h = 1)
```

Arguments

`x`	[cube] array of dimension `d` by `d` by `n`
`b`	[numeric] scale parameter, strictly positive
`h`	integer lag for excluding observations

Value

the value of the log objective function

`lscv_kdens_symmat` *Least square cross-validation for symmetric positive definite matrix kernels*

Description

Given a cube of sample observations (consisting of random symmetric positive definite matrices), and a vector of candidate bandwidth parameters `b`, compute the least square likelihood cross-validation criterion and return the bandwidth among the choices that minimizes the criterion.

Usage

```
lscv_kdens_symmat(x, b, h = 1L, kernel = "Wishart")
```

Arguments

`x`	array of dimension `d` by `d` by `n`
`b`	vector of candidate bandwidth, strictly positive
`h`	integer for the lag vector for determining which observation to exclude, any data point in a radius of `h`
`kernel`	string indicating the kernel, one of `Wishart`, `smnorm` or `smlnorm`.

Value

a list with arguments

- `lscv` vector of likelihood cross validation criterion
- `b` vector of candidate bandwidth
- `h` lag for leave-one-out
- `bandwidth` optimal bandwidth among candidates
- `kernel` string indicating the choice of kernel function

`lscv_kern_smlnorm` *Least square cross validation criterion for log symmetric matrix normal kernel*

Description

Finite sample h-block leave-one-out approximation to the least square criterion, omitting constant term. Only pairs that are $|i-j| \leq h$ apart are considered.

Usage

```
lscv_kern_smlnorm(x, b, h = 1L)
```

Arguments

`x`	[cube] array of dimension `d` by `d` by `n`
`b`	[numeric] scale parameter, strictly positive
`h`	[int] integer indicating the separation lag

Value

a double containing the log of the least square cross validation criterion

---

`lscv_kern_smnorm` *Least square cross validation criterion for matrix normal kernel*

Description

Finite sample h-block leave-one-out approximation to the least square criterion, omitting constant term. Only pairs that are $|i-j| \leq h$ apart are considered.

Usage

```
lscv_kern_smnorm(x, b, h = 1L)
```

Arguments

`x`	[cube] array of dimension `d` by `d` by `n`
`b`	[numeric] scale parameter, strictly positive
`h`	[int] integer indicating the separation lag

Value

a double containing the log of the least square cross validation criterion

---

`lscv_kern_Wishart` *Least square cross validation criterion for Wishart kernel*

Description

Finite sample h-block leave-one-out approximation to the least square criterion, omitting constant term.

Usage

```
lscv_kern_Wishart(x, b, h = 1L)
```

Arguments

`x`	[cube] array of dimension `d` by `d` by `n`
`b`	[numeric] scale parameter, strictly positive
`h`	separation vector; only pairs that are $|i-j| \leq h$ apart are considered

Value

a double containing the a vector of length two containing the log of the summands

---

`mgamma` *Multivariate gamma function*

Description

Given a vector of points `x` and an order `p`, compute the multivariate gamma function. The function is defined as

$$\gamma_p(x) = \pi^{p(p-1)/4} \prod_{i=1}^{p} \Gamma\{x + (1-i)/2\}.$$

Usage

```
mgamma(x, p, log = FALSE)
```

Arguments

`x`	[vector] of points at which to evaluate the function
`p`	[int] dimension of the multivariate gamma function, strictly positive.
`log`	[logical] if `TRUE`, returns the log multivariate gamma function.

Value

a matrix with one column of the same length as `x`

`realvar` *Realized variance of Amazon and SPY*

Description

Intraday realized covariances of the returns between the Amazon stock (`rvarAMZN`) and the SPDR S&P 500 ETF (`rvarSPY`) using five minutes data, for the period of September 13th, 2023 to September 12, 2024.

Usage

```
realvar
```

Format

A 2 by 2 by 250 array

Source

Anne MacKay

Examples

```
data(realvar, package = "ksm")
bopt <- bandwidth_optim(
 x = realvar,
 criterion = "lscv",
 kernel = "Wishart",
 h = 4L
 )
```

`Riccati` *Solver for Riccati equation*

Description

Given two matrices `M` and `S`, solve Riccati equation by iterative updating to find the solution $\mathbf{R}$, where the latter satisfies

$$\mathbf{R} = \mathbf{M}\mathbf{R}\mathbf{M}^\top + \mathbf{S}$$

until convergence (i.e., when the Frobenius norm is less than `tol`, or the maximum number of iterations `maxiter` is reached.

Usage

```
Riccati(M, S, tol = 1e-08, maxiter = 10000L)
```

Arguments

`M`	matrix
`S`	matrix
`tol`	double for tolerance
`maxiter`	integer, the maximum number of iterations

Value

a list containing

- `solution` matrix solution to Riccati's equation
- `error` numerical error
- `niter` number of iteration
- `convergence` bool indicating convergence (`TRUE`) if `niter` < `maxiter`

`rinvWishart` *Random matrix generation from the inverse Wishart distribution*

Description

Random matrix generation from the inverse Wishart distribution

Usage

```
rinvWishart(n, df, S)
```

Arguments

`n`	[integer] sample size
`df`	[double] degrees of freedom, positive
`S`	[matrix] a `d` by `d` positive definite scale matrix

Value

an array of dimension `d` by `d` by `n` containing the samples

---

`rmbeta2` *Random matrix generation from matrix beta type II distribution*

Description

This function only supports the case of diagonal matrices

Usage

```
rmbeta2(n, d, shape1, shape2)
```

Arguments

`n`	sample size
`d`	dimension of the matrix
`shape1`	positive shape parameter, strictly larger than $(d-1)/2$.
`shape2`	positive shape parameter, strictly larger than $(d-1)/2$.

Value

a cube of dimension `d` by `d` by `n`

`rmnorm` *Random vector generation from the multivariate normal distribution*

Description

Sampler derived using the eigendecomposition of the covariance matrix `vcov`.

Usage

```
rmnorm(n, mean, vcov)
```

Arguments

`n`	sample size
`mean`	mean vector of length `d`
`vcov`	a square positive definite covariance matrix, of the same dimension as `mean`.

Value

an `n` by `d` matrix of samples

Examples

```
rmnorm(n = 10, mean = c(0, 2), vcov = diag(2))
```

`rWAR` *Random matrix generation from first-order autoregressive Wishart process*

Description

Given a matrix of coefficients `M` and a covariance matrix `Sigma`, simulate `n` random matrices from a first-order autoregressive Wishart process by simulating from cross-products of vector autoregressions

Usage

```
rWAR(n, M, Sigma, K = NULL, order = 1L, burnin = 25L)
```

Arguments

`n`	sample size
`M`	matrix of autoregressive coefficients
`Sigma`	covariance matrix
`K`	integer, degrees of freedom
`order`	order of autoregressive process, only `1` is supported at current.
`burnin`	number of iterations discarded

Value

an array of size `d` by `d` by `n` containing the samples

References

C. Gourieroux, J. Jasiak, and R. Sufana (2009). The Wishart Autoregressive process of multivariate stochastic volatility, *Journal of Econometrics*, 150(**2**), 167-181, <doi:10.1016/j.jeconom.2008.12.016>.

Examples

```
M <- matrix(c(0.3, -0.3, -0.3, 0.3), nrow = 2)
Sigma <- matrix(c(1, 0.5, 0.5, 1), nrow = 2)
rWAR(n = 10, M = M, Sigma = Sigma, K = 5)
```

`rWishart` *Random matrix generation from Wishart distribution*

Description

Random matrix generation from Wishart distribution

Usage

```
rWishart(n, df, S)
```

Arguments

`n`	[integer] sample size
`df`	[double] degrees of freedom, positive
`S`	[matrix] a `d` by `d` positive definite scale matrix

Value

an array of dimension `d` by `d` by `n` containing the samples

---

`symmetrize` *Symmetrize matrix*

Description

Given an input matrix, symmetrize by taking average of lower and upper triangular components as $A + A^\top$.

Usage

```
symmetrize(A)
```

Arguments

`A`	square matrix

Value

symmetrized version of `A`

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