

Package ‘extr’

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Title Extinction Risk Estimation

Version 1.1.1

Description Estimates extinction risk from population time series under a drifted Wiener process using MLE and observation-error-and-autocovariance-robust estimators, with confidence intervals based on the w-z method.

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ext_di	<i>Extinction Risk Estimation for a Density-Independent Model</i>
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Description

Estimates demographic parameters and extinction probability under a density-independent (drifted Wiener) model. From a time series of population sizes, it estimates growth rate and variance using either a naive MLE variance estimator or an observation-error-and-autocovariance-robust (OEAR) estimator, then evaluates extinction risk over a horizon t^* with confidence intervals from the w - z method. In the observation-error-free setting, the naive w - z interval achieves near-nominal coverage, while OEAR can be more robust under additive observation error and unknown short-run autocovariance.

Usage

```
ext_di(
  dat,
  ne = 1,
  th = 100,
  alpha = 0.05,
  unit = "years",
  qq_plot = FALSE,
  formatted = TRUE,
  digits = getOption("extr.digits", 5L),
  method = c("naive", "oear")
)
```

Arguments

dat	Data frame with two numeric columns: time (strictly increasing) and population size. Column names are not restricted.
ne	Numeric. Extinction threshold $n_e \geq 1$. Default is 1.
th	Numeric. Time horizon $t^* > 0$. Default is 100.
alpha	Numeric. Significance level $\alpha \in (0, 1)$. Default is 0.05.
unit	Character. Unit of time (e.g., "years", "days", "generations"). Default is "years".
qq_plot	Logical. If TRUE, draws a QQ-plot of standardized increments to check model assumptions (naive method only). Default is FALSE.
formatted	Logical. If TRUE, returns an "ext_di" object; otherwise returns a raw list. Default is TRUE.
digits	Integer. Preferred significant digits for printing. Affects display only. Default is <code>getOption("extr.digits", 5)</code> .
method	Character. Variance estimation method. "naive" uses the MLE variance. "oear" uses the OEAR HAC long-run-variance estimator (AR(1) pre-whitening + Bartlett kernel). Default is "naive".

Details

Population dynamics follow

$$dX = \mu dt + \sigma dW,$$

where $X(t) = \log N(t)$ and W is a Wiener process. Extinction risk is

$$G = \Pr[T \leq t^* \mid N(0) = n_0, n_e, \mu, \sigma],$$

the probability that population size falls below n_e within t^* . Irregular observation intervals are allowed.

The function first estimates μ , then estimates a variance quantity according to method:

1. naive: uses the drifted-Wiener MLE variance (Dennis et al., 1991).
2. oear: uses an observation-error-and-autocovariance-robust (OEAR) HAC long-run-variance estimator with AR(1) pre-whitening and a Bartlett kernel (Hakoyama, in press).

For method = "oear", robustness to additive observation error relies on long-run-variance cancellation of differenced observation error (McNamara and Harding, 2004), while robustness to unknown short-run autocovariance comes from HAC estimation of long-run variance.

Extinction probability is then computed as $G(w, z)$ (Lande and Orzack, 1988), and confidence intervals are constructed using the w - z method (Hakoyama, in press).

Value

A list (class "ext_di" when formatted=TRUE) containing extinction-risk estimates and diagnostics. Core elements include Growth.rate, Variance, Unbiased.variance, lower_cl_s, upper_cl_s, linear_g, log_g, log_q, ci_linear_g, ci_log_g, ci_log_q, and data/input summaries (nq, xd, sample.size, unit, ne, th, alpha).

Variance is method-dependent (naive MLE for method="naive", OEAR HAC estimate for method="oear"). aic is reported only for method="naive" and is NA for method="oear". method_diag is NULL for naive and contains rho_tilde_pw and j for oear. lower_cl_s and upper_cl_s are chi-square based: exact under method="naive" model assumptions, and pragmatic plug-in approximations for method="oear".

References

- Andrews, D. W. K. (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica*, 59(3), 817-858.
- Dennis, B., Munholland, P.L., and Scott, J.M. (1991) Estimation of growth and extinction parameters for endangered species. *Ecological Monographs*, 61, 115-143.
- Hakoyama, H. Confidence intervals for extinction risk: validating population viability analysis with limited data. *Methods in Ecology and Evolution*. In press. Preprint, doi:10.48550/arXiv.2509.09965
- Lande, R. and Orzack, S.H. (1988) Extinction dynamics of age-structured populations in a fluctuating environment. *Proceedings of the National Academy of Sciences*, 85(19), 7418-7421.
- McNamara, J. M. and Harding, K. C. (2004). Measurement error and estimates of population extinction risk. *Ecology Letters*, 7(1), 16-20.
- Newey, W. K. and West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55(3), 703-708.

See Also

[statistics_di](#), [oear_sigma2_hac](#), [extinction_probability_di](#), [confidence_interval_wz_di](#), [print.ext_di](#)

Examples

```

# Example from Dennis et al. (1991), Yellowstone grizzly bears.
# Population is a running 3-year sum (3-year moving total) digitized from
# Fig. 5.
dat <- data.frame(Time = 1959:1987,
  Population = c(44, 47, 46, 44, 46, 45, 46, 40, 39, 39, 42, 44, 41, 40,
  33, 36, 34, 39, 35, 34, 38, 36, 37, 41, 39, 51, 47, 57, 47))

# Probability of decline to 1 individual within 100 years
ext_di(dat, th = 100)

# Probability of decline to 10 individuals within 100 years
ext_di(dat, th = 100, ne = 10)

# With QQ-plot
ext_di(dat, th = 100, ne = 10, qq_plot = TRUE)

# OEAR method
ext_di(dat, th = 100, method = "oear")

# Irregularly spaced observations: ML method
# Under the drifted Wiener process model, the ML method uses the exact
# likelihood for the observed time intervals; no interpolation is needed.
dat_irregular <- subset(dat, Time %in% c(1959, 1960, 1962, 1965,
  1969, 1974, 1980, 1987))

ext_di(dat_irregular, th = 100, ne = 10)

# A mostly complete annual series with a few missing observations
# can also be supplied using NA values. The ML method then uses the
# observed intervals between non-missing observations.
dat_with_na <- data.frame(
  Time = 1959:1975,
  Population = c(44, 47, NA, 44, 46, 45, 46, NA, 39, 39,
  42, 44, NA, 40, 33, 36, 34)
)

ext_di(dat_with_na, th = 100, ne = 10)

# OEAR method with mild missingness
# Unlike the ML method above, OEAR is an effective-diffusion approximation
# based on the long-run variance of increments. Mild missingness is
# acceptable, but avoid using method = "oear" when missing observations
# are numerous or observation intervals are highly heterogeneous.
ext_di(dat_with_na, th = 100, ne = 10, method = "oear")

```

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